The influence of binary stars on dwarf spheroidal galaxy kinematics

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ABSTRACT
This paper presents the first Monte Carlo simulation of binary stars in dwarf spheroidal galaxies that explores the effect of varying the distributions of the orbital parameters on the observed velocity dispersion. Our approach enables us to estimate the percentage of binary stars that would be recognized as such over the course of repeated observations. From the results presented here, it is possible to assess the relative merits of repeated observations and smaller measuring errors when trying to detect binary stars. In order to make these calculations, a more general method has been adopted than methods used previously. Using the method presented in this paper it should be possible for all observers of stars in dwarf spheroidal (dSph) galaxies to compare the numbers of binary stars that they have observed with any distribution of parameters for the binary orbits. Our conclusion is that for orbital distributions similar to those measured for the solar neighbourhood, the contribution of binary stars to the velocity dispersion is small. The number of binary stars that have been detected by observers of dwarf spheroidal galaxies is also consistent with these distributions. The truth of this latter conclusion is, however, dependent on the exact size of the measuring errors and the number of years over which observations have been made. Orbital parameters, very different from those measured for the solar neighbourhood, would be required for the binary stars to be making a significant contribution to the velocity dispersions that have been measured for dSph galaxies. In particular more smaller period orbits with higher mass secondaries would be required. The shape of the velocity distribution may help to resolve this issue when more data become available since, in general, the scenarios producing a larger apparent dispersion have a velocity distribution that deviates more clearly from Gaussian.

Key words: binaries: general – galaxies: kinematics and dynamics.

1 INTRODUCTION
A binary star orbits its companion in an elliptical orbit, its velocity changing as it goes. This velocity depends on the masses of the stars, the period of the orbit and the eccentricity of the orbit. The orbit of the stars that we would observe has the centre of mass of the two stars as one of the foci. The velocity that we observe at a particular time depends on the position of the star in its orbit and the orientation of the orbit with respect to an observer. If every star observed was on an identical orbit, but at a different point in that orbit, a range of different velocities would be observed, the average velocity being zero. The standard deviation of these velocities is the dispersion produced by this orbit. A particular range of different orbits therefore contributes a specific amount to the velocity dispersion that would be observed. The size of the contribution of these binary stars to the dispersion can be calculated if the number of stars with each set of orbital parameters is known. It is this value that was estimated by the simulation described in this paper.

1.1 Previously published results
Aaronson & Olszewski (1987) made the first simulations for binary stars in dSph galaxies, taking the mass and period distributions from Galactic studies by Mathieu (1983), and choosing a primary mass of 0.8 $M_\odot$. They chose the phase,
intrinsic dispersion in their results.

Mateo et al. (1993), Suntzeff et al. (1993), and Vogt et al. (1994) used similar simulations to estimate the effect of binaries in Carina, Sextans and Leo II dSph galaxies respectively. They took the period from a distribution that is uniform in log space, which is consistent within the errors of the data from various studies, including Duquennoy & Mayor (1991). The mass was taken from a uniform distribution, the primary mass being 0.8 $M_{\odot}$. The inclination was taken from a cosine distribution, and the eccentricity and phase were chosen at random, with the distribution again not stated. The sample sizes for the simulations were similar in size to the observed sample reported in the papers (between 17 and 33 stars), and they repeated the simulation for differing intrinsic velocity dispersions and binary fractions, making 1000 independent trials each time. They calculated the standard deviation and the biweight (Beers, Flynn & Gebhardt 1990) of the resulting velocities. For small intrinsic dispersions of the order of 2 km s$^{-1}$, which are expected for dSph galaxies containing no dark matter, a binary fraction of 0.2 was required in order to produce an apparent velocity dispersion of the size observed in dSph galaxies ($> 6$ km s$^{-1}$). They also noticed a difference between the standard deviation and biweight measurements, and suggested that because this was not observed in their observations, it was an indication that binaries did not contribute significantly to the observed dispersion. This would require the binary star fraction in dSph galaxies to be considerably less than that observed in the solar neighbourhood.

1.2 Observations of binary stars

Hargreaves et al. (1994a, 1994b) have made multi-epoch observations of 18 stars in Sextans and Ursa Minor. Of these, two stars show velocity variations which indicate that they may be binary stars. The variation of one of these stars is a far more significant detection than the other. If both these stars are binary stars, the observed binary fraction was 0.11 over two years of observations. It is important to know what actual binary fraction this represents, since if less than half of the binaries were observed in two years, this would imply a binary fraction of greater than 0.2. Other observations of stars in dSph galaxies have obtained between 4 and 12 yr of repeat measurements of 63 stars in Sculptor, Fornax, Ursa Minor and Draco. These observations have found a binary fraction of between 0.1 and 0.16 (Mateo 1994). For Draco, where there are 24 stars for which there are up to five observations of each, four appear to be binaries (Mateo 1994). Our simulation was designed to place an estimate on the fraction of binary stars that would be observed over a certain period of observations, as well as to calculate the contribution binary orbits make to the measured velocity dispersion.

2 Details of the model

The model was constructed to examine the velocity distribution caused by the orbits of binary stars. The model made a Monte Carlo simulation, choosing binary orbits with parameters chosen at random from empirical and theoretical distributions for a large number of stars, and then evolved each star around its orbit. From this it was possible to ascertain what fraction of binary stars would be identified over the course of a certain number of equally spaced observations, given a certain velocity above which a velocity difference would become apparent to an observer. The velocity distribution obtained from evolving the stars around their orbits could be used to calculate the velocity dispersion caused by the binary stars.

The parameters of the binary orbits were chosen randomly from different distributions in the model. The velocity that would be measured by an observer was calculated for equal time intervals all the way around each orbit. In this way, a distribution of velocities was obtained for each orbit. Of these velocities, 100 were chosen at random from the set of velocities for each star and written to file 1, ensuring that the distribution for each orbit represented in the file had equal weight. Thus, file 1 contained the velocity distribution for the set of binary orbits.

It was assumed that a specific difference in velocity, called the threshold velocity, could be detected between two velocity measurements made by the observer: the value depended on the assumed measuring errors. If the difference in velocity between time intervals, measured along the line of sight, was equal to or greater than this value, then the velocities concerned were marked. All the velocities that appeared in file 1, and were not marked in this way, were written to file 2. Therefore file 2 contained the velocity distribution for the binary orbits which would not be identified by the assumed measuring errors. The fraction of the binary stars that would be identified was given by the ratio of the number of velocities in files 1 and 2. The velocity dispersion of the binary distribution ($\sigma_b$ in equation 12) was equal to the standard deviation of the velocities in file 1, whereas the velocity dispersion that would be obtained if the stars were thrown out of the sample when identified as belonging to binary systems was the standard deviation of the stars in file 2. The standard deviations of the data in files 1 and 2 were consistent under repetition for given parameter distributions, provided that a sufficient number of stars were used in the sample.

The orbital parameters that need to be considered are: the masses of the two stars; the period of the orbit; the minimum approach distance of the stars; the ellipticity of the orbit; the inclination; the phase; and the position of the pericentre with respect to the observer.

2.1 The distributions of the orbital parameters

The best estimate for the distributions of the orbits of binary stars comes from the solar neighbourhood sample observed by Duquennoy & Mayor (1991), so it is these distributions that have been used to define the orbital parameters. The primary stars in the orbits of that study were solar mass G dwarfs. Although the masses of the stars observed in the dSph galaxies are fairly close to the masses of these dwarfs, it may well be the case that the orbital distributions discovered
are not applicable in the very different conditions of a dSph galaxy. The simulations that used these orbit distributions, assuming the radius of the primary to be 10 and 30 $R_0$, are termed DM10 and DM30 respectively in the rest of this paper. To compare this with previous simulations, the same distributions used by Mateo et al. in their simulations were also used (hereafter Ma). The secondary mass distribution calculated by Kroupa, Tout & Gilmore (1993), which rises rather than falls towards low masses, was also used. In these simulations, the DM distributions were used for the other parameters apart from mass: the simulations are called KTG10 and KTG30.

2.1.1 Mass

Carbon stars are the brightest stars in dSph galaxies, but these are few in number and have a high probability of being velocity variables (McClure 1984). The next brightest stars, occupying the tip of the giant branch, are the K giants and it is observations of these that are used to calculate the velocity dispersions. Given that the stellar populations in dSph galaxies are predominantly of intermediate to old age, these stars will have mass of about 0.8 $M_\odot$, so this is the value of the primary in our simulations. Fig. 1 shows that there is very little difference in the results of the simulation when considering a primary mass of between 0.6 and 1.0 $M_\odot$. The secondary mass distribution found by Duquennoy & Mayor is given by

$$\mathcal{P}(M_2) \propto \exp\left[ -\frac{(M_2 - 0.23)^2}{0.42} \right],$$

(1)

where $M_2$ is allowed to vary between 0.05 $M_\odot$ and the mass of the primary.

Mateo et al. also took the primary mass to be 0.8 $M_\odot$, but their secondary masses were taken from a uniform distribution with masses varying between 0.05 and 0.8 $M_\odot$.

Kroupa, Tout & Gilmore's (1993) mass distribution has the following form:

$$\mathcal{P}(m) = \begin{cases} 
0.03m^{-1.3} & 0.08 \leq m \leq 0.5 \\
0.019m^{-2.2} & 0.5 \leq m \leq 1.0 \\
0.019m^{-2.7} & 1.0 \leq m < \infty
\end{cases}$$

(2)

2.1.2 Period

The Duquennoy & Mayor (1991) period distribution is consistent with the estimate of Kroupa, Tout & Gilmore (1990). It is Gaussian in log space, and has the form

$$\mathcal{P}(|\log(P_{\text{days}})|) \propto \exp\left[ \frac{(P_{\text{days}} - 4.8)^2}{2.3} \right]$$

(3)

where $P_{\text{days}}$ is the period in days. No maximum or minimum bounds were imposed on this distribution.

The distribution used by Mateo et al. was uniform in the logarithm of the period.

2.1.3 Ellipticity

Here, the ellipticity, $e$, is defined to be $e = (1 - b/a)$, where $b$ and $a$ are the minor and major axes of the orbit respectively.

Duquennoy & Mayor found that the ellipticity obeyed the following distributions.

$$\mathcal{P}(e) = \begin{cases} 
0.0 & \text{period} < 11 \text{ d} \\
\exp\left[ -\frac{(e - 0.3)^2}{0.16} \right] & 11 \text{ d} \leq \text{period} < 1000 \text{ d} \\
2e & \text{period} > 1000 \text{ d}
\end{cases}$$

(4)

Mateo et al. (1993) stated that they had used ellipticities in the range 0.5 to 1.0, so, for the Ma model, we chose the ellip-

![Figure 1. The velocity dispersion obtained for different primary masses for model DM10.](http://mnras.oxfordjournals.org/ at University Library on March 15, 2014)
ticity from a flat distribution between these limits. However, this was a misprint (Mateo, private communication). The true limits were 0.0 and 0.5. This should have little effect on the results, as the orbits with very high ellipticity are not present in our simulation (see Section 4.1.1).

2.1.4 Angles

Fig. 2 shows the orbit of a binary star round its centre of mass. Viewed from the Earth, the star moves in an ellipse in the plane perpendicular to our line of sight. The inclination, \(i\), is the angle between the orbital plane and the viewing plane. For spherical symmetry, the normal to the viewing plane must be evenly distributed over the sphere, implying that the distribution of the orbital inclinations is proportional to the sine of the inclination.

\[
\mathcal{P}(i) \propto \sin i. 
\]  
\[(5)\]

The angle \(w\) is the angle between the ascending node, \(M\), and the periastron of the orbit. As in the diagram, the phase, \(v\), is the angle between \(w\) and the current position of the star, taken in the direction shown. Both \(w\) and \(v\) were allowed to vary between 0° and 360°, so, because of symmetry, \(i\) was allowed to vary between 0° and 180°. The angle \(w\) was chosen from a uniform distribution, and was a constant parameter of the orbit. The phase, \(v\), however, is the angle that varies with time, so that, although its initial value was chosen from a uniform distribution, its value thereafter varied in accordance with Kepler’s 2nd law.

2.2 Radius cut-off

Orbits of very low period or high ellipticity may not be physically possible, since they can result in too close an encounter between the two stars in the binary. To estimate this minimum distance, a simple gravitational Roche-lobe radius estimate was used.

\[
M_2 = M_1 \times \frac{a_p^3 (a_p - R)}{R^3 (2a_p - R)}. 
\]  
\[(6)\]

Here \(a_p\) is the distance between the stars at periastron, and \(R\) is the radius of the primary star of mass \(M_1\), \(M_2\) being the maximum mass of the secondary before Roche-lobe overflow occurs. The radius of the giant stars in dSph galaxies is not well known, as the evolution of stars in low metallicity environments such as that found in dSph galaxies is not understood in detail; therefore a range of between 10 and 30 \(R_\odot\) was used in the simulations, which should cover the possibilities. This is a difference between the binaries in dSph galaxies and the binaries with solar mass primaries observed by Duquennoy & Mayor in the solar neighbourhood: some of the orbits that existed around the primaries when they were main-sequence stars should no longer exist, so it may be expected that the fraction of binary stars will be lower than the fraction observed in the solar neighbourhood.

2.3 Velocity

From the period, the semimajor axis of the real orbit, rather than that with respect to the centre of mass, is given by

\[
a^3 = (M_1 + M_2) T^2. 
\]  
\[(7)\]

where \(a\) is the semimajor axis in au, \(T\) is the period in yr and \(M_1\) and \(M_2\) are the masses in \(M_\odot\).

The line of sight velocity observed for a particular binary star with mass \(M_1\) at some phase, \(v\), is given by

\[
V = 2\pi \frac{1.49598 \times 10^{11}}{365.25 \times 3600} \frac{M_2 \sin i}{\sqrt{a(M_1 + M_2)}} \left[ \cos(v + w) + \frac{e \cos w}{\sqrt{1 - e^2}} \right]. 
\]  
\[(8)\]

Here the velocity is in \(\text{m s}^{-1}\), and the semimajor axis, \(a\), is in au. The ellipticity, \(e\), is \(\sqrt{1 - (b/a)^2}\).

2.4 Time

In order to calculate the velocity that would be observed at equal time intervals around the orbit using equation (8), it was necessary to calculate the change in the phase resulting from a change in time.

Fig. 3 shows the geometry of the orbit. The secondary, \(M_2\), sits at the focus of the ellipse orbited by \(M_1\). The phase, \(v\), is also called the true anomaly and \(E\) is the eccentric anomaly. Geometry and Kepler’s second law can be used to derive the following relationships between these two angles and time.

\[
\tan \frac{E}{2} = \sqrt{1 - e^2} \tan \frac{v}{2}, 
\]  
\[(9)\]

\[
E - \sin E = \frac{2(\tau)}{T}. 
\]  
\[(10)\]

Here \(\tau\) is the time since the stars were last at periastron.


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Hence we obtain $E$ from $v$ using equation (9), iterate to find the new $E$ from equation (10), and then use equation (9) once more to calculate the new $v$. In this way, and using equation (8), the velocity can be calculated at every stage of the orbit.

2.5 Fraction of binary stars identified

There were some binary stars that would never be identified because the range of their orbital velocities was small compared with the measuring error. These were the stars for which the difference between the maximum and minimum value of the velocity from equation (8) was less than the threshold velocity at which the difference would appear significant to the observer (this is explained for specific examples in Section 3). The maximum and minimum values were obtained by evaluating the equation at $v = -w$ and $v = \pi - w$. For these stars, the orbit was divided into 100 equal time-steps and each velocity produced was written to files 1 and 2.

The time difference between measurements in the simulations was taken to be one year. Therefore each star that was not already catered for by the criterion of the previous paragraph was evolved around its orbit, recording the velocity at one-year time intervals. For periods of less than 100 yr, 1001 yearly velocities were taken. For periods greater than this, the number of orbital revolutions was reduced to save computing time. For periods between 100 and 1000 yr, velocities were calculated five times around the orbit, and for those between 1000 and 10000 yr, two orbits' worth of velocities were calculated. Then all the differences between consecutive velocities were calculated, thus obtaining 1000 velocity differences for periods of less than 100 yr.

The velocities were marked for which the star would be identified as having a binary orbit, over the course of the observations, if observations were started from the position on the orbit associated with that velocity. Therefore another parameter was used, which was the number of years over which observations were conducted. A velocity was marked if any of the subsequently obtained velocity differences, or their sum, was greater than the threshold velocity. Then 100 velocities were chosen at random with equal probability from the sample for each star, writing all 100 to file 1 and only those that were not marked to file 2.

For periods greater than 10 000 yr, a slightly different approach was adopted. For such large periods, the only portion of the orbit at which the star might possibly be identified as a binary was close to periastron, so 1001 velocities were calculated symmetrically about periastron, and the phase angles, about which the velocity differences were sufficiently large to be detected over the years of observation, were calculated. Then the velocity was calculated at 100 equal time-steps right around the orbit (i.e. each time-step went 100th of the way around the orbit) and marked those that fell between the calculated phase points. These velocities were written to the files in the same way as the velocities for the other orbits.

For the periods of less than 10 000 yr, the probability of recognizing a particular binary star orbit over the observing period was equal to the number of marked velocities divided by the number of observations in total. For the larger periods, the probability was equal to the number of velocities within the calculated phase range where the binary could be identified, divided by the number of time-steps contained in an orbit. For a time-step of one year, this denominator is just the orbital period in years.

The average of the probabilities for all the stars is equal to the fraction of stars that would be recognized as having binary orbits.

3 RESULTS

Figs 4, 5, 6, 7 and 8 show the results for the simulations. Each figure contains two plots. The upper plot shows the percentage of the binaries that would be identified for a certain threshold velocity. The different lines are for yearly observations spanning 2, 10 and 50 yr. The solid line on the lower plot shows the standard deviation of the distribution arising from binary stars with the orbital parameters chosen: the dotted lines show the standard deviation of the distributions once the identified binary stars have been removed.

For observations with an error for each velocity measurement of $\sigma_{\text{err}}$, the threshold velocity for a binary star to be detected was $3\sqrt{2}\sigma_{\text{err}}$. For the observations of Hargreaves et al. (1994a, 1994b), the error per velocity was close to 2 km s$^{-1}$. Aaronson & Olszewski (1987) took 4 km s$^{-1}$ as the threshold velocity in their simulation, whereas the average error on the velocities of Pryor, Olszewski & Armandroff (1995) was 3.6 km s$^{-1}$, giving a threshold velocity of 15.3 km s$^{-1}$. Threshold velocities of 1, 4, 8.5, 15.3, and 21.2 km s$^{-1}$ were used, as shown in the plots, to cover the range of possible observations.

The simulation was made using 10 000 stars, since it was at this level that repeat simulations became easily recognizable; these results were reproducible, so the simulation was convergent. The one standard deviation of the percentage of identified binaries taken from repeat simulations was about 0.5 per cent, whereas that for the velocity dispersion was about 0.06 km s$^{-1}$. Table 1 shows the standard deviation of the binary star distribution required for binary stars to be making a significant contribution to the measured dispersion.
in dSph galaxies; dispersions owing to binary orbits alone are required to be greater than about 6 km s\(^{-1}\) in order to explain the observed dispersion, assuming a low mass-to-light ratio. The results from the simulations from the chosen parameter range show that the highest dispersion caused by the binary orbits alone is about 3 km s\(^{-1}\). We conclude that either the velocity dispersions that have been observed are largely unaffected by binary stars, or that the orbital parameters, which were after all taken from galactic observations, are inappropriate for dSph galaxies.

Comparing the results from the Ma, DM10 and DM30 models, it can be seen that the percentage of binaries detected in model Ma was considerably larger than that in the other two cases. This is largely due to the fact that the orbits in the Ma model have a maximum period of 10 000 yr. The periods in the DM model have no such upper bound, and have about 30 per cent of periods above 10 000 yr. These long periods alone produce only a small dispersion, and as they can only be detected during the few years close to periastron, they only slightly increase the total number of binary stars identified. When an extra 30 per cent of stars are added analytically to the Ma model using equations (11) and (12), the percentages and dispersions (approximating the dispersion of the long periods alone to be negligible), lie somewhere between the results from the 2 DM models. The KTG mass distribution which rises towards the low mass end, does, as expected, produce lower percentages of binaries identified and lower velocity dispersions.

### 3.1 Models that produce large velocity dispersions

Fig. 9 shows the standard deviation of the velocity distribution of the DM10 model at different periods. The periods were chosen from a uniform distribution within a small range around the average value shown on the plot. It was not sensible to take single periods, because the dispersion would be greater at integer multiples of the time between measurements (because if the large periastron velocities were measured once, then they were measured every orbit); this does not represent the real-life situation where observations are not taken at exact intervals. Only 1000 rather than 10 000 stars were used for each of these simulations since, with such a limited period distribution, the velocity distribution was quicker to converge.

Periods of less than 5 yr can produce large standard deviations of greater than 6 km s\(^{-1}\). As a demonstration of this, simulations were made with the orbital parameters of Mateo et al. and period ranges of 0.5 to 10 yr, and 0.5 to 100 yr. These produced dispersions of 5.5 and 4.3 km s\(^{-1}\), respec-
tively. This all implies that keeping the other parameters as before, only for the situations where the binary fraction is close to 1.0, and the periods are almost all below 10 yr, can the velocity dispersion produced by the binary stars be sufficient to account for what is observed. For a threshold velocity of 4 km s\(^{-1}\) (equivalent to a one sigma error of 0.9 km s\(^{-1}\) on each velocity), one would expect to see more than 70 per cent of the binaries in 10 yr for periods below 10 yr, requiring a far higher binary fraction observed than the 10–20 per cent actually observed. This is true, even allowing for the fact that not all these stars have been observed for 10 yr.

Fixing the mass of the secondary to be equal to the mass of the primary also increases the standard deviation of the velocity distribution, the DM10 model producing a dispersion of over 6 km s\(^{-1}\) (see Fig. 10). Here, only 30 per cent of the binaries would be identified in 10 yr, for a threshold velocity of 4 km s\(^{-1}\), which is much more in line with the observations.

As we have seen earlier, taking very high ellipticity orbits can produce high dispersions; however, these orbits are not physically possible and therefore are not allowed using the cut-off radius method.

4 MAIN DIFFERENCES BETWEEN THE SIMULATIONS AND PREVIOUS WORK

4.1 Binary fraction

It is straightforward to calculate the size of the observed dispersion from the binary velocity dispersion, the binary fraction, and the intrinsic velocity dispersion. Assuming that the two dispersions both have a mean of zero,

\[ \sigma_a^2 = (1 - f) \sigma_b^2 + \sigma_b^2, \]  

(11)

where

\[ \sigma_b^2 = \sigma_d^2 + \sigma_i^2, \]  

(12)

here \( \sigma_a \) is the observed dispersion, \( \sigma_i \) is the intrinsic velocity dispersion, \( \sigma_d \) the calculated dispersion owing to a binary fraction of 1.0, and \( f \) is the binary fraction.

In order to compare our results with those of the papers mentioned in Section 1.1, we adopted the following procedure. Equations (11) and (12) were used to subtract the effect of the intrinsic dispersion and binary fraction leaving only the dispersion owing to the binary orbits. Fig. 11 shows the results from Mateo et al. (1993) adjusted in this manner as the dotted line, with our simulation using their distribu-
4.1.1 Period and radius cut-offs

The range of ellipticity quoted by Mateo et al. (1993) was 0.5 to 1.0 and the lower bound on the period range was 0.5 or 1 yr. The authors now state this as a misprint, the true limits on the ellipticity distribution being 0.0 and 0.5 (Mateo, private communication). However, it is still important to point out the problems of including very high ellipticity orbits, since these ellipticities could lead to unrealistic orbits in which the two stars in the binary overlap during the orbit. Fig. 12 shows how the measured velocity dispersion varies for a fixed ellipticity. Here the period, taken from a distribution which is uniform in the logarithm of the period, ranges from 0.5 to 10 yr. To avoid the problems caused by this effect, the simulation described in this paper uses a minimum approach distance for the two stars rather than a minimum period and maximum ellipticity. This is a more physically accurate picture as it is the expansion of the stars after their main sequence lifetime which is thought to cause the loss of some of the lower period orbits.

5 ANALYSIS OF THE RESULTS

5.1 Comparison with the observations

If the parameters defining binary star distributions in the Galaxy and dSph galaxies are the same, except for some loss of close orbits owing to the expansion of the primary in the older population, then the effect of binaries on the measured velocity dispersion is small. If so, then there must be some other explanation for the large velocity dispersions that have been measured in dSph galaxies. When all the multi-epoch observations from dSph galaxies are added together, a binary fraction of between 0.1 and 0.16 is observed, with 4 to 12 yr of measurements (Mateo 1994). Aaronson & Olszewski (1987) have between 5 and 10 (roughly yearly) measurements for their stars, and have detected a binary fraction of between 0.1 and 0.2. These results have not been fully published, so there remain uncertainties about the exact answers, but they do claim a measuring error of about 1 km s⁻¹, therefore a threshold velocity of about 4 km s⁻¹ should be suitable for analysing their results. Duquennoy & Mayor
(1991) found a binary fraction of 0.6 for the solar neighbourhood solar mass stars.

Considering the range of the results from the DM10 and DM30 models, between 12 and 24 per cent of the binaries should be identified in 5 to 10 yr, given a threshold velocity of 4 km s$^{-1}$ (see Fig. 13). If 60 per cent of the stars are binary stars, this means that we should actually identify 7 to 14 per cent of the stars as binary stars. If the threshold velocity is 8.5 km s$^{-1}$ (equivalent to a measuring error of 2 km s$^{-1}$), we would expect between 4 and 10 per cent of the binaries to be identified as such. The percentage of binaries that has been
Figure 12. The velocity dispersion for the Ma model with periods between 0.5 and 10,000 yr, for different values of the ellipticity $e = \sqrt{1 - (b/a)^2}$.

Figure 13. Variation of the percentage of binaries detected with number of years of observation. The top plot shows the results from the DM10 model with threshold velocities of 1, 4, and 8.5 km s$^{-1}$ (solid, dotted and dashed lines respectively), and the lower plot presents results of the DM30 model, adopting the same threshold velocities.

detected is a little on the high side (see the previous paragraph), but the discrepancy may well be caused by an underestimate of the measuring errors; for example, this could occur if there were broad wings on the error distribution. We correct for the fact that 28 per cent of the binaries in DM30 and 15 per cent in DM10 were rejected when compared with the DM1 model, where the radius of the primary was 1 R$_{\odot}$, because the minimum separation of the stars fell below the cut-off, and assume that these ex-binaries are still ‘normal’ stars in the sample. Then we conclude that the binary fraction should be 0.4 or 0.5, rather than 0.6. This results in a predicted percentage of binaries detected in 5–10 yr, with a threshold velocity of 4 km s$^{-1}$, of 5 to 12 per cent, which is more divergent from the observations. From the DM models, with a threshold velocity of 4 km s$^{-1}$, we should expect never to identify between 40 and 50 per cent of the binary stars, for however many years we observe (Table 2). However, the dispersion caused by this fraction is very small, of the order of 1 km s$^{-1}$ (see Figs 5 and 6). From the shape of the velocity distribution formed by the binary orbit velocities alone, it appears that one would not reject the Gaussian distribution hypothesis with only 20 stars, but would in most cases with 40 stars. Therefore, with a binary fraction of 0.6, we would expect no divergence from a Gaussian shape to be detected at the level of our current observations.

Aaronson & Olszewski’s observations of Draco have detected a binary fraction of 0.17 with up to five observations at roughly yearly intervals and with measuring errors of about 1 km s$^{-1}$. This again is slightly on the high side.

For the results by Hargreaves et al. (1994a, 1994b) one or two binaries out of 18 stars with multi-epoch observations in Sextans and Ursa Minor (6 to 11 per cent) may have been found with two years of observation and a measuring error of 2 km s$^{-1}$. The DM models, and a binary fraction of 0.6, predict that 1.8 to 3.3 per cent of the binaries should have been
identified. We would, therefore, have expected to see 0 or 1 binary star. However, several of the multi-epoch measurements have more than one observation at each of the two epochs, as is the case for the strongest binary candidate. This leads to a considerably higher probability of identifying a binary star owing to the effective decrease in measuring error that results from the combination of several observations.

The results from the Ma model, restricting the period to various ranges (Fig. 11), suggest that nearly all the binary stars which are identified within 10 yr have periods of less than 100 yr. This result was obtained by considering the sample with periods between 0.5 and 10 000 yr. Since we know that the distribution is uniform in the logarithm of the period, we can calculate the percentages of the total number of binary stars identified in the more restricted period ranges. The result is a negligible difference in the percentage between periods of 100 and 10 000 yr.

For the distribution to be such that the binary stars have a significant effect on the measurement of the velocity dispersion, we would require the sample to be biased towards lower periods and higher masses. In this case we would expect to detect between 30 and 70 per cent of the binaries in 10 yr of observation, assuming a threshold velocity of 4 km s\(^{-1}\) (Table 2). Thus, the observations imply a binary fraction between 0.14 and 0.50. This requires a dispersion of about 10 km s\(^{-1}\) or more for the binary stars to have a significant effect on the observed dispersion (Table 1). In these cases, the distributions of the orbital parameters of the binary stars are very different from the distributions of Galactic binary stars that have been detected through observations.

Simulations using a very small primary mass radius can also produce large dispersions: if the radius of the primary star were as small as 5 R\(_{\odot}\), then dispersions greater than 5.5 km s\(^{-1}\) could be produced, while stars of 1 R\(_{\odot}\) could produce a dispersion of close to 10 km s\(^{-1}\) (Fig. 14).

The percentages of binary stars detected with the KTG models are only slightly less than those for the DM models. Thus it is impossible, from these results, to rule out the KTG mass function, which rises towards lower masses. If anything, it is slightly more in tune with the observations.

### 5.2 The shape of the velocity distribution

The shape of the velocity distribution varies depending on the model chosen. Table 2 shows the sample size at which the distributions would probably be rejected as being Gaussian at the 3\(\sigma\) level by a \(K\)-\(S\) test. The distributions have broader wings than a Gaussian distribution. These sample sizes do not, however, reflect the number of stars that we would need to observe before recognizing a component owing to binary stars in an observed distribution. This is because the simulation has not taken into account the contribution to the distribution of the intrinsic velocity dispersion owing to the mass of the galaxy.

A simple procedure was completed to illustrate the situation. Assuming the intrinsic velocity distribution of dSph galaxies is Gaussian in form, randomly chosen Gaussian deviates were added to the sample of velocities obtained from the binary star simulation. The intrinsic dispersion was chosen such that the total dispersion of the resulting sample was 6 km s\(^{-1}\). \(K\)-\(S\) tests were then conducted for different sample sizes drawn from the new distribution.

For the DM10 distribution 40 stars were sufficient to reject the Gaussian hypothesis. This distribution had a dispersion of 3.8 km s\(^{-1}\) which was caused by binary star velocities alone. When we added a Gaussian deviate from an intrinsic dispersion of 4.6 km s\(^{-1}\) to each star (making 6 km s\(^{-1}\) in total, using equations 11 and 12), the \(K\)-\(S\) test did not reject the Gaussian hypothesis until samples contained as many as 1000 stars. The addition to each sample of 67 per cent more stars, which had a velocity from the intrinsic dispersion but no binary component, required the intrinsic dis-
The velocity dispersion caused by just the binary orbits, with parameters corresponding to the solar neighbourhood, was suppressed to be 5.2 km s\(^{-1}\). This sample, with a binary fraction of 0.6, required about 5000 stars before it was rejected at the 3\(\sigma\) level by the K–S test.

This result was compared with a simulation for which the dispersion caused by the binary stars alone is larger. In this case, the DM10 simulation was performed, restricting the periods of the orbits to less than three years, the periods being drawn from a uniform distribution. The dispersion caused by the binary stars only was 6.2 km s\(^{-1}\) and samples of about 220 stars were required before this distribution was rejected in the K–S test. We made the same calculation as described in the previous paragraph for this sample, but only conducted the experiment for a binary fraction of 0.6, because a binary fraction of 1.0 would require no contribution from an intrinsic dispersion. The intrinsic dispersion required by a binary fraction of 0.6 to make a total dispersion of 6 km s\(^{-1}\) was 3.2 km s\(^{-1}\). In this case, about 500 stars were required for the Gaussian hypothesis to be rejected. Fig. 15 shows these results for the two simulations.

In the light of these examples, it seems unlikely that there should be any clear evidence for binary stars in the shape of the velocity distributions which have been obtained from dSph galaxies, because the largest sample sizes are about 80 stars. As an illustration, we have combined the data from the observations reported by Hargreaves et al. (1994a, 1994b, 1995) for the Sextans Ursa Minor and Draco dSph galaxies. Each velocity distribution was normalized to a dispersion of 1 km s\(^{-1}\) and then a K–S test was performed on the whole sample. This sample contained 74 stars and included the first epoch measurements from both suspected binary stars in Sextans. The probability from the K–S test was 0.8. Fig. 16 shows the combined sample with the Gaussian function overlaid.

6 CONCLUSION

The velocity dispersion caused by just the binary orbits, with parameters corresponding to the solar neighbourhood, was
small compared with the large velocity dispersions observed in dSph galaxies. The percentage of binary orbits that would be identified depends on the number of years of observation, and on the precision of the velocity measurements. However the simulations, which use orbital distributions derived from real observations, predict the identification of percentages of binary stars that are only slightly less than those actually observed.

To produce larger dispersions, more binary orbits with a mixture of lower periods, higher mass secondaries, or primaries with radii smaller than 10 R_☉ are required. It is difficult to produce a velocity dispersion much above 6 km s^{-1} without requiring restriction of the orbits to periods below about 5 yr. For velocity dispersions from binary stars of the order of 6 km s^{-1} to significantly modify the overall observed dispersion, a binary fraction of close to 1.0 would be required. From observations of such a population spanning 10 yr, about 30 per cent of the binaries should be identified with such distributions. The observations are now slightly on the low side for this scenario, at 10–20 per cent, even accounting for the fact that not all of these stars have been observed for 10 yr.

At present it seems likely that some of the stars that observers have assumed to be binaries are erroneous detections, as a consequence of an underestimate of their measurement errors. Continuing high-precision observations, when analysed using the results of this work, will be able to quantify the contribution of the binary stars to the observed velocity dispersion. Samples of greater than about 500 stars, with single-epoch measurements, would be required to define the kinematic distribution function, but precise data for a few stars over a long time interval should be able to quantify the true significance of binary stars in the dynamics of dSph galaxies.

ADDENDUM

Private communication from M. Mateo.

It has come to my attention that there is in fact an error in the code used to calculate the observed velocities for the binary stars in Mateo et al., Suntzeff et al., and Vogt et al. The net effect is that the velocities are overestimated significantly. The effect on the dispersions is to inflate a 5 km s^{-1} dispersion to about 6.8 km s^{-1} for a binary frequency of 30 per cent. When the error in the program is corrected, there is almost a negligible change in the dispersion when the initial value is 5 km s^{-1}.

REFERENCES


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